1) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$F(x) = 3x^2 - 2$

$\int_{-1}^{1} 3x^2 - 2$

$\Delta x = \frac{1-(-1)}{n} = \frac{2}{n}$

+1

$x_i = -1 + \frac{2i}{n}$

$F\left(-1 + \frac{2i}{n}\right) = 3\left(-1 + \frac{2i}{n}\right)^2 - 2$

$= \left(3\left(1 - \frac{4i}{n} + \frac{4i^2}{n^2}\right) - 2\right)\frac{2}{n}$

$= 3 - \frac{12i}{n} + \frac{12i^2}{n} - 2)\ \frac{2}{n}$

$\lim_{n\to\infty} \sum \frac{6}{n} - \sum \frac{24}{n^2} \sum i + \frac{24}{n^3} \sum i^2 - \frac{4}{n} \sum 1$

$= \frac{6}{n} - \frac{24}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{24}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) \cancel{n} - 4$

$= 2 - 12\ \frac{(n^2)}{n^2} \to 1 + 4\ \frac{(2n^3)}{n^3} \to 2$

$4 \cdot 2$

$\Rightarrow 2 - 12 + 8$

$= 10 - 12$

$\boxed{= -2}$

Alfredo Gonzalez 8/5/2014

QUESTION #2.

BRAYAN CASTANEDA

$$\int \frac{dx}{\sqrt{(x-7)(x-9}} = \int \frac{dx}{\sqrt{(x-8)^2-1}}$$ COMPLETE THE SQUARE

$$= \int \frac{\sec\theta \tan\theta \, d\theta}{\sqrt{\sec^2\theta - 1}}$$

LET $x-8 = \sec\theta$

$dx = \sec\theta \tan\theta \, d\theta$

$$= \int \frac{\sec\theta \tan\theta \, d\theta}{\sqrt{\tan^2\theta}}$$

$\sec^2\theta - 1 = \tan^2\theta$

$$= \int \frac{\sec\theta \cancel{\tan\theta} \, d\theta}{\cancel{\tan\theta}}$$

$$= \int \sec\theta \, d\theta$$

$$= \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \boxed{\ln\left|x-8 + \sqrt{(x-7)(x-9)}\right| + C}$$

$\sqrt{\tan^2\theta} = \sqrt{(x-7)(x-9)}$

$\tan^2\theta = (x-7)(x-9)$

$\tan\theta = \sqrt{(x-7)(x-9)}$

Scott Myatt

3. $$\int_0^{\frac{1}{11}} \frac{1}{(121x^2+1)^{3/2}}\,dx = \int_0^{\frac{1}{11}} \frac{1}{\left(\sqrt{(11x)^2+1}\right)^3}\,dx$$

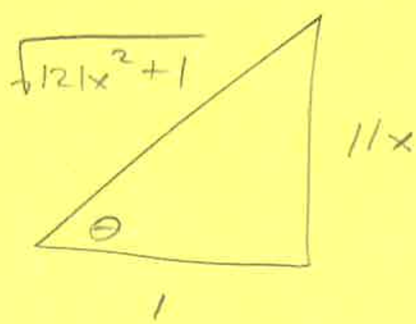
$$\tan\theta = 11x$$

$$x = \frac{\tan\theta}{11} \qquad dx = \frac{1}{11}\sec^2\theta\, d\theta$$

$$= \frac{1}{11}\int_0^{\frac{1}{11}} \frac{\sec^2\theta\, d\theta}{\left(\sqrt{\sec^2\theta}\right)^3} = \frac{1}{11}\int_0^{\frac{1}{11}} \frac{\sec^2\theta\, d\theta}{\sec^3\theta}$$

$$= \frac{1}{11}\int_0^{\frac{1}{11}} \frac{1}{\sec\theta} = \frac{1}{11}\int_0^{\frac{1}{11}} \cos\theta\, d\theta$$

$$\frac{1}{11}\sin\theta \Big|_0^{\frac{1}{11}}$$



$$= \frac{1}{11}\left(\frac{11x}{\sqrt{121x^2+1}}\right)\Big|_0^{\frac{1}{11}}$$

$$= \frac{1}{11}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{11\sqrt{2}} = \boxed{\frac{\sqrt{2}}{22}}$$

Scott Myatt
8-4-14
MATH 76

4: $$\int \frac{\sqrt{y^2-100}}{y}\,dy, \quad y > 10$$

$$= \int \frac{\sqrt{(y)^2-(10)^2}}{y}\,dy$$

$$y = 10\sec\theta$$
$$dy = 10\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sqrt{(10\sec\theta)^2-(10)^2}}{10\sec\theta}(10\sec\theta\tan\theta\,d\theta)$$

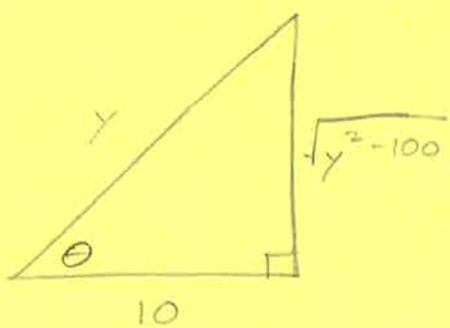
$$= \int \frac{\sqrt{100\sec^2\theta-100}}{10\sec\theta}(10\sec\theta\tan\theta\,d\theta) = \int \frac{\sqrt{100}\cdot\sqrt{\sec^2\theta-1}}{10\sec\theta}10\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\cancel{10}\sqrt{\tan^2\theta}}{\cancel{10}\sec\theta}10\sec\theta\tan\theta\,d\theta = \int \frac{10\cancel{\sec\theta}\tan^2\theta}{\cancel{\sec\theta}}\,d\theta$$

$$= \int 10\tan^2\theta\,d\theta = 10\int(\sec^2\theta-1)\,d\theta$$

$$= 10(\tan\theta-\theta)+C$$

$$y = 10\sec\theta$$
$$\frac{y}{10} = \sec\theta$$
$$\frac{y}{10} = \frac{1}{\cos\theta}$$
$$\sec^{-1}\left(\frac{y}{10}\right) = \theta$$



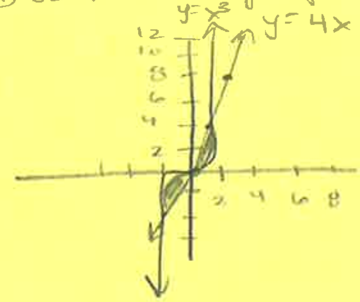
$$\boxed{10\left(\frac{\sqrt{y^2-100}}{10}-\sec^{-1}\left(\frac{y}{10}\right)\right)+C}$$

⑤ Find the area enclosed by the given curves.

Ephia

$y = x^3$, $y = 4x$

(*) sketch a graph



(*) set equations equal to one another to find intersection points.

$x^3 = 4x$

$x^2 = 4$

$x = 2, \cancel{-2}$

$$\text{Area} = \int_0^2 4x - x^3\,dx \times 2$$

$$\text{Area} = \frac{4x^2}{2} - \frac{x^4}{4}\Bigg|_0^2 \times 2$$

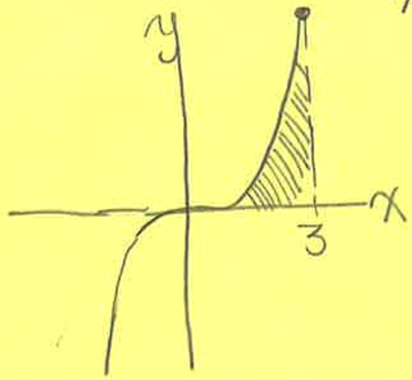
$$\text{Area} = \left[\frac{4(2)^2}{2} - \frac{(2)^4}{4}\right] - \left[\frac{4(0)^2}{2} - \frac{(0)^4}{4}\right] \times 2$$

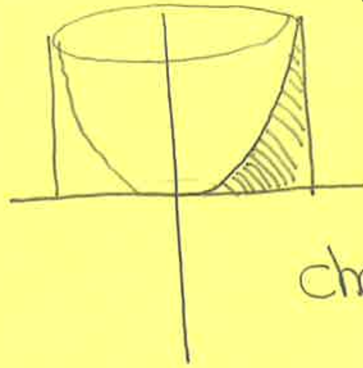
$$\text{Area} = \Big[[8-4] - [0-0]\Big] \times 2$$

$$\boxed{\text{Area} = (4) \times 2 = \boxed{8}}$$ Ⓐ

#6

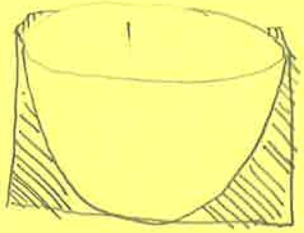
$x^3=3$





washer/disk method

about x axis
change to terms in y



$$\pi\int_0^{27}(3-\sqrt[3]{y})^2\,dy=\pi\int_0^{27}(3^2-y^{2/3})\,dy$$

$$=\pi\left[9y-\frac{y^{5/3}\cdot 3}{5}\right]\Bigg|_0^{27}$$

$$=\pi\left[\left(9(27)-\frac{(27)^{5/3}\cdot 3}{5}\right)-(0)\right]$$

$$=\pi\left[243-\frac{729}{5}\right]$$

$$=\frac{486\pi}{5} \quad B.$$

about y axis
keep in terms of x

Shell method

$$=2\pi\int_0^3(x)(x^3)\,dx$$

$$=2\pi\int_0^3(x^4)\,dx$$

$$=2\pi\left[\frac{x^5}{5}\right]\Bigg|_0^3$$

$$=2\pi\left[\left(\frac{3^5}{5}\right)-\left(\frac{0^5}{5}\right)\right]$$

$$=2\pi\left[\left(\frac{243}{5}\right)\right]=\frac{486\pi}{5} \quad B.$$

Miguel Bueno

Sophia Medina

7 Find length of curve
$y = (9-x^{2/3})^{3/2}$ from $x=1$ to $x=27$

$L = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$

$y' = \frac{3}{2}(9-x^{2/3})(-\frac{2}{3}x^{-1/3})$

$(y')^2 = \frac{9-x^{2/3}}{\sqrt[3]{x^2}}$

$L = \int_1^{27} \sqrt{\frac{x^{2/3}}{x^{2/3}} + \frac{9-x^{2/3}}{x^{2/3}}}\, dx$

$L = \int_1^{27} \sqrt{\frac{3^2}{(x^{1/3})^2}}\, dx$

$L = \int_1^{27} \frac{3}{x^{1/3}}\, dx \quad L = 3\int_1^{27} x^{-1/3}\, dx$

$L = (3)\frac{3}{2}x^{2/3}\Big|_1^{27}$

$\frac{81}{2} - \frac{9}{2}$

$= \boxed{36}$ m

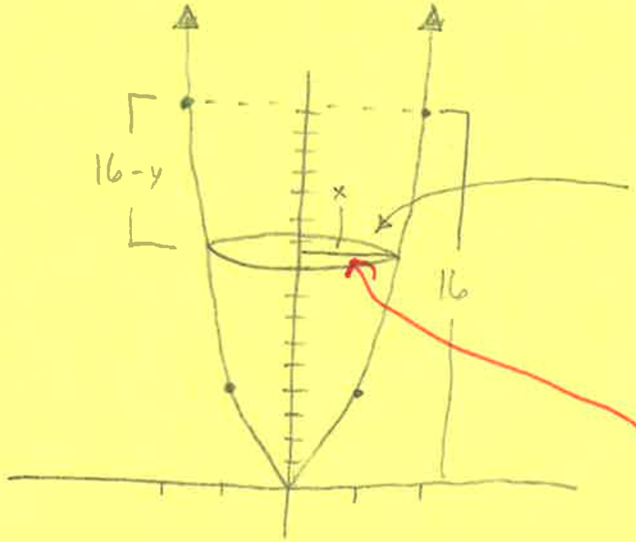
Daily Quiz #7: Practice Final

Kyle Hayter
8-5-14

8!

$y = 4x^2, \quad 0 \le x \le 2, \quad y\text{-axis}$

$\rho g = 9{,}800 \ N/m^3$



Find the area of this circle:

$y = 4x^2$

$\frac{y}{4} = x^2$

$x = \sqrt{\frac{y}{4}}$

πr^2

$\pi \left(\sqrt{\frac{y}{4}}\right)^2$

Then use the formula for work: $W = \rho g \int_0^b A \cdot (h-y)\,dy$

$W = 9{,}800 \int_0^{16} \pi \left(\sqrt{\frac{y}{4}}\right)^2 \cdot (16-y)\,dy$

$W = 9{,}800\pi \int_0^{16} \frac{y}{4} \cdot (16-y)\,dy$

$W = 9{,}800\pi \int_0^{16} \frac{16y}{4} - \frac{y^2}{4}\,dy$

$W = 9{,}800\pi \int_0^{16} 4y - \frac{1}{4}y^2\,dy$

$W = 9{,}800\pi \cdot \left[\frac{4y^2}{2} - \frac{1}{12}y^3\right]\Big|_0^{16}$

nice!

$W = 9{,}800\pi \left[2y^2 - \frac{y^3}{12}\right]\Big|_0^{16}$

$W = 9{,}800\pi \left[\left(2(16)^2 - \frac{(16)^3}{12}\right) - 0\right]$

$W = 9{,}800\pi \left[512 - 341.3333\right]$

$W = 9{,}800\pi \left[170.6667\right]$

$W = 1{,}672{,}533.66\pi$

$\boxed{W = 5{,}254{,}418 \text{ J}}$

Ruby Rodriguez

#9) $\sum_{n=1}^{\infty} \frac{15}{(5n-4)(5n+1)}$

nice!

$\frac{A}{(5n-4)}^{(5n-4)(5n+1)} + \frac{B}{(5n+1)}^{(5n+1)(5n-4)} \rightarrow A(5n+1) + B(5n-4) = 15$

$\rightarrow 5An + A + 5Bn - 4B = 15$

$\rightarrow (5A+5B)n = 0 \rightarrow 5A+5B=0 \rightarrow 5A+5B=0$

$A - 4B = 15 \qquad -5(A-4B)=15 \qquad -5A+20B=-75$

$25B = -75$

$\overline{25}$

$B = \boxed{-3}$

$A - 4(-3) = 15$

$A + 12 = 15 - 12$

-12

$\boxed{A=3}$

Telescoping!

$= 3\sum_{n=1}^{\infty}\left(\frac{1}{5n-4} - \frac{1}{5n+1}\right) = 3\left(\frac{1}{1} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{11}}\right) + \left(\cancel{\frac{1}{11}} - \cancel{\frac{1}{16}}\right)$

$+ \left(\cancel{\frac{1}{16}} - \frac{1}{21}\right)$

Because they all cancel except for the first term

$= 3(1) = 3$ Series converges

#10 $\sum_{k=1}^{\infty} \frac{(k!)^8}{(8k)!}$

Trevor Patrick

Ratio test

$$\frac{\frac{((k+1)!)^8}{(8k+8)!}}{\frac{(k!)^8}{(8k)!}} = \frac{((k+1)!)^8 (8k)!}{(8k+8)!\,(k!)^8}$$

$$\lim_{k\to\infty} \frac{[(k+1)\cancel{k!}\,(k+1)\cancel{k!}\,(k+1)\cancel{k!}\,(k+1)\cancel{k!}\,(k+1)\cancel{k!}\,(k+1)\cancel{k!}\,(k+1)\cancel{k!}\,(k+1)\cancel{k!}](8k)!}{(8k+1)!\ \cancel{k!}\ \cancel{k!}\ \cancel{k!}\ \cancel{k!}\ \cancel{k!}\ \cancel{k!}\ \cancel{k!}\ \cancel{k!}}$$

$$\lim_{k\to\infty} \frac{(k+1)^8 \cancel{(8k)!}}{(8k+8)(8k+7)(8k+6)(8k+5)(8k+4)(8k+3)(8k+2)(8k+1)\cancel{(8k)!}}$$

$$\lim_{k\to\infty} \frac{(k+1)^8}{(8k+8)(8k+7)(8k+6)(8k+5)(8k+4)(8k+3)(8k+2)(8k+1)} = \frac{O(k^8) + \text{lower order}}{O((8^8)(k^8)) + \text{lower order}}$$

$$\frac{O(k^8)}{O(k^8)8^8}$$

$$\lim_{k\to\infty} \frac{1}{8^8} < 1$$

$\boxed{\text{convergent}}$

Guadalupe Calvillo

$\boxed{11}$ $\displaystyle\sum_{n=1}^{\infty} \frac{8}{n\sqrt{n+8}}$ let $b_n = \dfrac{1}{n^{3/2}}$

$p > 1 \therefore b_n$ Converges.

$$= \lim_{n\to\infty} \frac{\dfrac{8}{n\sqrt{n+8}}}{\dfrac{1}{n^{3/2}}}$$

$$= \lim_{n\to\infty} \frac{8}{n\sqrt{n+8}} \cdot \frac{n\sqrt{n}}{1}$$

$$= \lim_{n\to\infty} \frac{8n\sqrt{n}}{n\sqrt{n+8}}$$

$$= \lim_{n\to\infty} \frac{8\sqrt{n}}{\sqrt{n+8}} \frac{\left(\frac{1}{\sqrt{n}}\right)}{\left(\frac{1}{\sqrt{n}}\right)}$$

$$= \lim_{n\to\infty} \frac{8}{\sqrt{1+\cancel{\frac{8}{n}}}} \to 0$$

$$= \frac{8}{\sqrt{1+0}}$$

$= 8 > 0 \therefore \displaystyle\sum_{n=1}^{\infty} \frac{8}{n\sqrt{n+8}}$ Converges by the Limit Comparison test and p-series

✓ nice!

12) $\sum_{n=1}^{\infty} \frac{2n^2+7}{n^5+5n+2}$ $\frac{a_n}{b_n}$ Daryl Dancy Jr.

Use Limit Comparison Test

$$\frac{\frac{2n^2+7}{n^5+5n+2}}{\frac{1}{n^3}}$$

$\frac{1}{n^3}$ → Because its 1) convergent P-series

2) n^3 flips up to cancel like terms

Good!

$$\frac{(2n^2+7)n^3}{n^5+5n+2} = \frac{\boxed{2n^5} + 7n^3}{\boxed{n^5} + 5n + 2} \quad \frac{\theta(2\cancel{n^5})}{\theta(\cancel{n^5})} \Rightarrow 2 > 0$$

Convergent because it's comparible to convergent P-series

13. $\sum_{n=1}^{\infty} \frac{\ln(5n)}{n^4}$

Aaron Singh

$$\lim_{b\to\infty} \int_1^b \frac{\ln(5x)}{x^4}\,dx$$

let $u = \ln 5x$ $\quad dv = \frac{1}{x^4}dx$

$du = \frac{1}{x}dx \quad v = -\frac{3}{x^3}$

$$\frac{-3\ln(5x)}{x^3}\Big|_1^\infty + \int_1^\infty \frac{3}{x^4}\,dx$$

$$\frac{-3\ln 5x}{x^3}\Big|_1^\infty + 3\left(-\frac{3}{x^3}\right)\Big|_1^\infty$$

$$\frac{-3\ln 5x}{x^3}\Big|_1^\infty - \frac{9}{x^3}\Big|_1^\infty$$

$$\left(\frac{-3\ln(5(\infty))}{\cancel{\infty^3}} + \frac{3\ln 5(1)}{1}\right) - \left(\cancel{\frac{9}{\infty^3}} + 9\right)$$

$=$

$$\frac{3\ln 5}{9} < 1$$

Converges

Daryl Dancy Jr.

14) $\sum_{n=1}^{\infty} (-6)^{-n} = \left(-\frac{1}{6^n}\right) = \left(-\frac{1}{6}\right)^n$

$\sum_{n=1}^{\infty} \left(-\frac{1}{6}\right)^n$ $= \left(-\frac{1}{6}\right)$ is the "r", so by the geometric series test,

$|r| < 1 = \left|\cancel{-}\frac{1}{6}\right| < 1$

$\boxed{= \text{Converges!}}$

And sum is:

$\frac{1}{1+\frac{1}{6}} - 1 = \frac{6}{7} - 1 = -\frac{1}{7}$

Conditional or Absolute???

Use Absolute Convergence Test (ACT)

$\sum_{n=1}^{\infty} \left|-\frac{1}{6}\right|^n = \left(\frac{a}{1-r}\right) - 1$ $r = \frac{1}{6}$ for Absolute value.

$\left[\frac{1}{1\cancel{+}\left(\frac{1}{6}\right)}\right] - 1 = \left(\frac{1}{\frac{5}{6}}\right) - 1 = \frac{6}{5} - 1 = \boxed{\frac{1}{5}}$

$\cancel{\frac{6}{7} - 1 = \frac{6}{7} - \frac{7}{7} = -\frac{1}{7}}$

$\sum \left|\left(-\frac{1}{6}\right)\right|^n = \sum \left(\frac{1}{6}\right)^n$

~~Take the absolute value:~~ $\cancel{\left|-\frac{1}{7}\right| = \frac{1}{7}}$

$r = \frac{1}{6}$

$\boxed{\text{Converges Absolutely}}$ ✓ yes →

15.) $\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2 9^n}{(2n+1)!}$

Aaron Singh

Ratio Test:

$$\lim_{b\to\infty} \left| (-1)^{n+1} \cdot \frac{(n+1)!^2\, 9^{n+1}}{(2n+3)!} \cdot \frac{\cancel{(2n+1)!}}{\cancel{(-1)^n}\cancel{(n!)^2}\, \cancel{9^n}} \right|$$

$(2n+3)(2n+2)\cancel{(2n+1)!}$

$$\lim_{b\to\infty} \left| \frac{(-1)(n+1)(n+1)\cdot 9}{(2n+3)(2n+2)} \right|$$

$$= \lim_{b\to\infty} \left| \frac{-9(n+1)\cancel{(n+1)}}{(2n+3)\cdot 2\cancel{(n+1)}} \right| = \lim_{b\to\infty} \left| \frac{-9(n+1)}{4n+6} \right|$$

$$= \lim_{b\to\infty} \left| \frac{-9n-9}{4n+6} \right|$$

$$\left| -\frac{9}{4} \right| > 1$$

Diverges

#16 $\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n^2+1} - n\right)$

Alt. series. test.

$\left|(-1)^n\left(\sqrt{n^2+1} - n\right)\right| = \sqrt{n^2+1} - n$

$\lim_{n\to\infty} \sqrt{n^2+1}$

note: $f(x) = \sqrt{x^2+1} - x$

$f'(x) = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x - 1$

$= \frac{x}{\sqrt{x^2+1}} - 1 \longrightarrow$ Always negative when $x > 0$

$\frac{x}{\sqrt{x^2+1}} < 1$

$\therefore$ f(x) is decreasing and obviously positive on $[1, \infty)$

$\therefore$ Fits criteria for Alt-series test.

$\lim_{n\to\infty} \sqrt{n^2+1} - n \cdot \frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n} = \frac{\cancel{n^2}+1-\cancel{n^2}}{\sqrt{n^2+1}+n}$

$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}+n} = 0$. $\therefore$ converges.

I will skip next step, but by the ~~integral~~ test $\int_1^b \sqrt{x^2+1} - x\,dx \cancel{\to} \infty$

(conditional convergence)

Ruby Rodriguez

#17) $\sum_{k=1}^{\infty} \sin\left(\frac{8}{k^5}\right)$

$\lim_{k\to\infty} \frac{a_k}{b_k} = L$

a_k

$$\frac{\sin x}{x} \xrightarrow{x \to 0} 1$$

$\lim_{k\to\infty} \sum_{k=1}^{\infty} \frac{\left(\frac{\sin(8/k^5)}{\cos(8/k^5)}\right)}{\left(8/k^5\right)} \leftarrow b_k \rightarrow \frac{\sin(8/k^5)}{\left(\frac{8}{k^5}\right)\cos(8/k^5)}$

1

$= 1 \cdot \left(\frac{1}{\cos(8/k^5)}\right) \quad 1 > 0$

nice!

p series $\left(\frac{1}{k^p}\right)$

So by the limit comparison test the series converges because $1 > 0$.

191

Use root test to determine if it converges/Diverges

$$\sum_{n=1}^{\infty}\left(\frac{\ln(n)}{5n+2}\right)^n = \lim_{n\to\infty}\left|\frac{\ln(n)}{5n+2}\right| = \frac{\infty}{\infty}$$

Use l'Hopitals

$\frac{\frac{1}{n}}{5}$ as $n \to \infty$ $\frac{5}{n} \to 0$

Therefore...

$$\lim_{n\to\infty}\left|\frac{\ln(n)}{5n+2}\right| = 0$$

It Converges by L'Hopital's Rule or speed of convergence.

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{5} \longrightarrow 0$$

Andrew Shaffer

#20

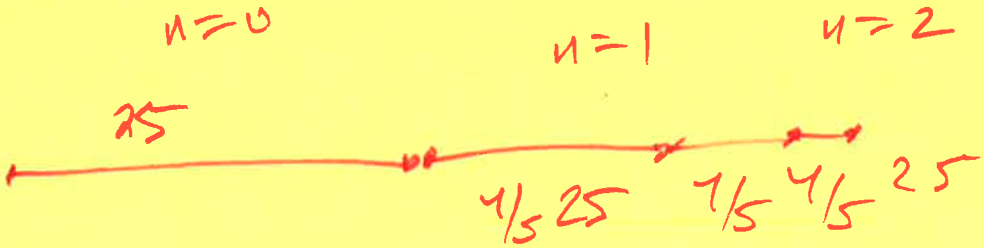
[illegible]

the answer is "D"

$$25\left(\frac{4}{5}\right)^n = \frac{25}{1-\frac{4}{5}} = \frac{25}{\frac{1}{5}} = 125$$

or

$$\frac{4}{5} = 5 \quad 5 \times 25 = 125$$



$$\text{Total Distance} = \sum_{n=0}^{\infty} 25\left(\frac{4}{5}\right)^n$$

$$= \frac{25}{1-4/5} = \frac{25}{1/5} = 125 \checkmark$$